# Optimal Selection of Selective Mining Unit Size for Geostatistical Modeling

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**Abstract**

*The calculation of mineral resources and ore reserves from a block model requires the choice of a block of selective mining unit (SMU) size. Each block is assigned a grade or a distribution of grades. The resources/reserves are calculated from these block values. Selecting an appropriate SMU size requires consideration of the mining equipment, bench height, blasthole sampling, grade control practice, and affect of dilution. The use of large mining equipment will preclude use of an SMU size that is smaller than the volume of material that can be extracted by the equipment. Dilution is another important consideration that will affect the tonnes and grade of ore. Conventional grade control practice traditionally involves using information from blasthole samples and on-site visual inspections to refine the ore-waste boundary. We do not attempt to address the use of visual controls on grade control. Clearly, if there are visual controls in the pit they must be considered.*

*This paper proposes a methodology to determine the optimal SMU size to match actual production. Actual production is simulated on a representative area by simulating the collection of blasthole data and the consequent grade control. Then, the geostatistical resource estimation procedure is implemented for a range of SMU block sizes and the SMU size that gives a reasonable match to the actual production is recommended. The optimal SMU size will yield an estimate of tonnes of ore and grade of ore that is close to the actual production. An example is shown to illustrate the methodology. Different types of dilution and the affect of grade control data sampling on the SMU size are also discussed.*

## Introduction

The conventional definition of the selective mining unit (SMU) is *the smallest volume of material on which ore waste classification is determined* (Sinclair and Blackwell, 2002). The reality is more complex. It is impractical and impossible to freely select an SMU of ore in the midst of waste just as it is impossible to freely reject an SMU of waste in the midst of ore. Nevertheless, even large bulk mining equipment may have the ability to mine within a couple of meters of a boundary if the conditions are favorable. The SMU size depends on a number of different factors, including the mining equipment size, the mining method to be used, the direction of mining, and the depositional environment of the orebody.

Our unconventional definition of the selective mining unit (SMU) is *the block model size that would correctly predict the tonnes of ore and diluted head grade that the mill will receive with anticipated grade control practice*. Mutual exclusion of ore and waste material means that correctly predicting the tonnes of ore entails correct prediction of the tonnes of waste. This size must somehow be related to the ability of the equipment to select material, but it is also based on the data available for classification (blastholes and/or dedicated grade control drilling), the procedures used to translate that data to mineable dig limits, and the efficiency with which the

mining equipment excavates those dig limits. Numerous sources of dilution must also be accounted for including internal dilution due to grade variability within the SMU, external dilution resulting from geological/geometric contacts, and operational dilution that accounts for production errors, pressures and schedule demands.

Conventional grade control practice uses information from blasthole samples and on-site visual inspections to refine the ore-waste boundary. We do not attempt to address the use of visual controls on grade control. Clearly, if there are visual controls in the pit they must be considered. A common way of translating blasthole data to dig limits is the *outline-and-average* method where ore or waste regions are delineated by a polygon that implicitly accounts for the equipment. Kriging is sometimes used to improve on the border between ore and waste. Simulation and loss functions have gained limited use in further refining the boundaries between ore and waste. The available data is a clear limitation to the resolution with which we can pick limits. Dedicated grade control sampling or closer spaced smaller diameter blastholes provide some refinement, but a cost-benefit analysis must be performed.

In practice, the tonnes/grade of ore that the mill receives is the result of a classification procedure with many subjective factors. The mill certainly does not receive the values in a long- or medium-term block model. There is much more information at the time of mining and the blocks are never freely and perfectly selected in any case. It would be impractical with existing software and computational resources to create many realizations and simulate the classification procedure on the multiple high-resolution models. We must consider the reality of block modeling for the present time. Thus, we are forced to choose an SMU size for the reporting of resources/reserves.

Block estimates may be considered deterministically as is done in the vast majority of kriged block models. The modern probabilistic paradigm is to calculate a probability of waste, probability of ore, and grade of ore for each SMU block by simulation. The probabilities are associated to proportions, for example, 8 out of 10 blocks with an 80% probability of ore are considered as ore, therefore we add 80% of each block's tonnage to the ore tonnes and 20% to the waste tonnes.

Estimates of what the mill will produce and the amount of waste are based on our chosen SMU size. We require a method for selecting an SMU size that yields the same tonnage and grade as conventional grade control practice. This paper proposes a method that uses conditional simulation to generate multiple realizations of grades, based on which tonnes of ore and grade of ore can be calculated for a range of SMU sizes. Comparison against the results of conventional grade control practice gives the optimum SMU size. Some examples are shown to illustrate the methodology.

**Proposed Methodology**
The proposed approach for SMU size selection uses information from both the anticipated grade control practice and realizations from conditional simulation of the long term resource model (or the kriged model if that is the chosen method). The idea is to compare the tonnage and grade obtained from grade control with the tonnage and grade obtained after processing the realizations at a series of different SMU sizes. The first step is to choose a reasonably large and representative production area, $A$, that likely represents quarterly production. Multiple areas could be chosen and/or different areas could be chosen within different rock types.

The following procedure is undertaken for each representative area, $A$:

1. Simulate a high-resolution realization accounting for all geological controls, trends, and available data in the region. The resolution of the realization should be 1/3 to 1/10 of the anticipated blasthole spacing and the bench height.

2. Sample the realization with blasthole grades at the anticipated spacing. Dedicated grade control drilling could be considered at this step if that is planned.

3. Simulate the grade control practice to arrive at ore/waste dig limits. The idea is to mimic the actual grade control that will be implemented in the mine. The alternatives include outline-and-average, blasthole kriging, and simulation combined with profit maximization or loss minimization. It is difficult to anticipate all of the operational considerations; however, it may be a good idea to err on the side of conservatism. For example, blasthole kriging is a good starting point for this exercise to be followed by the best simulation and profit maximization. The slight conservatism of the blasthole kriging will be offset by operational dilution.

   The ore/waste dig lines can be further smoothed (some erosion/dilation algorithm) to account for dilution considerations and the fact that they cannot be mined exactly. Then, the ore/waste dig lines are used with the reference high-resolution grades to calculate the expected tonnes of ore, $T_o$ - all tonnes flagged as ore regardless of grade, and the grade of ore, $z_o$ - the average grade of all material flagged as ore. The idea will be to determine the SMU size that matches these reserves.

4. Use the drillhole samples, which are available at a coarser spacing than the blasthole data, to generate multiple realizations of grades at a high resolution. These realizations will match the sparser exploration drilling and the geological controls in an approximate manner. It is important not to be too optimistic, for example, the geological boundaries cannot be frozen for the entire exercise.

5. Choose a range of possible SMU sizes and block average all the realizations to the SMU size by simply calculating a density-weighted average of the grades. Apply the cutoff grade to all realizations and for all locations, $\mathbf{u} \in A$, and obtain the probability to be above the cutoff grade ($P(Z(\mathbf{u}) \geq z_c)$), probability to be below the cutoff grade ($P(Z(\mathbf{u}) < z_c)$), and the average grade above the cutoff grade ($z_o(\mathbf{u})$). Calculate the tonnes of ore, $T_o$, and the grade of ore, $z_o$.

6. Plot the results of both the grade control and the simulation approach in a two graphs: (1) tonnes of ore versus SMU size, and (2) average grade of ore versus SMU size. In each graph, the conditional simulation results are plotted to yield a functional relationship, while the grade control values provide a single true value that plots as a horizontal line. The optimal SMU size is the size at which these two lines intersect. Figure 1 shows a schematic illustration of these graphs, showing two different functional relationships that depend on whether the cutoff grade is above or below the mean grade. The optimal SMU size is shown for the latter case.

Many of the considerations previously mentioned do not intervene directly in this procedure. Moreover, there is a risk that the results are too optimistic because the same geostatistical parameters are used for both the reference realization, grade control and the simulation for resource assessment. The procedure could be refined to account for more factors, and result in a slight increase in the observed SMU size.
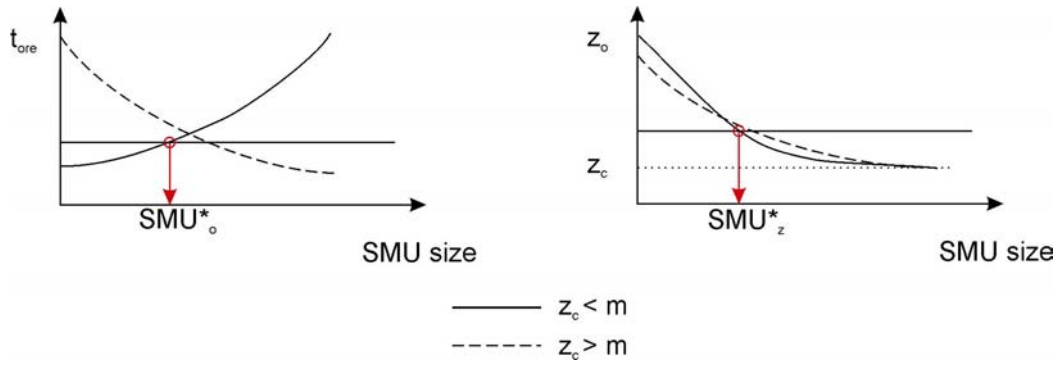
Figure 1 Schematic illustration of graphs constructed for optimal SMU size selection: tonnes of ore versus SMU size (left), and grade of ore versus SMU size (right). Note that in each graph the optimal SMU size is selected for the case where the cutoff grade is below the mean grade.

**Example**

A synthetic example is used to illustrate the methodology. A reference data set is generated via unconditional simulation with a histogram and a variogram. The variogram is arbitrarily chosen with a maximum continuity direction at 35 degrees azimuth with the following model:

$$\gamma(\mathbf{h}) = 0.05 + 0.55 Exp_{a_{hmax}} = {}_{1100}(\mathbf{h}) + 0.40 Sph_{a_{hmax}} = {}_{\infty} (\mathbf{h})$$

$$\begin{array}{cclcccl} a_{hmin} & = & 900 & & a_{hmin} & = & 2000 \\ a_{vert} & = & 10 & & a_{vert} & = & 10 \end{array}$$

The reference data are generated at a resolution of 1m x 1m x 5m that spans an area of 600m x 600m. The block height corresponds to an arbitrarily small 5m bench. Assuming a specific gravity of 2.7, this volume corresponds to a quarterly production volume of just under 5 million tonnes (at nominally 55 000 tonnes/day). A cutoff grade of 5.0% is applied to each location and a reference ore-waste map is obtained. Figure 2 shows the reference data histogram, variogram and maps.

**Conventional Grade Control.** Blasthole data are sampled from this reference map at nominally 10m x 10m spacing. A small random component is added to the sampled data to mimic potential sampling errors in the field. These blasthole data are then used to perform estimation of the grades at a fine 2m x 2m x 5m grid using ordinary kriging (Figure 3). For the two maps, the cutoff grade is applied to show only those values strictly above the cutoff grade. An ore-waste contact outline is drawn corresponding to the trimmed map of estimates because it gives a less noisy approximation of the boundary between ore and waste.

This ore-waste outline is applied to the reference grade map to determine the mill's production (Figure 4). For an assumed specific gravity of 2.7, there are 2.426 million tonnes of ore at an average grade of 7.35%, and there are 2.434 million tonnes of waste. These become the reference or base values for $T_o$ and $z_o$ for checking against the following simulation approach.

**Simulation-based Grade Control.** Exploration drillholes are sampled from the reference map at approximately 50m x 50m spacing. These samples are used to construct a conditional simulation model for small 5m x 5m block sizes. The ten realizations generated are then block averaged to a range of possible SMU sizes: 10x10, 15x15, 20x20, 25x25, 30x30, 40x40, and 50x50 . Figure 5 shows an example of the block averaged results for one realization at the eight different SMU sizes.

For each block model, the cutoff grade is applied and the probability and grade of ore are calculated. Figure 6 shows the maps for the probability and average grade above

cut off for SMU sizes 10x10, 20x20, 30x30 and 40x40. Using these maps, the tonnes and grade of ore are calculated and compared against the reference values using the ore-waste contacts (Figure 7). Based on the tonnes of ore, the optimal SMU size is approximately 20m x 20m; however, based on the grade of ore, the choice of SMU size should be 6m x 6m.

Figure 7 shows that for an approximate SMU size of 20m x 20m blocks, the tonnes of ore and waste would match the mill's production. For this same SMU, the grade of ore would be 7.22%, while the mill would produce 7.35% (a percentage difference of 2%). Alternatively, if we tried to match the grade of ore, then the optimal SMU size is approximately 6m x 6m blocks, resulting in an estimated 2.37 million tonnes of ore, instead of the reference production of 2.42 million tonnes of ore (a difference of 4%). For this example, the appropriate choice in SMU size will match the tonnes of ore, since the mismatch between the estimated and actual grade of ore differs by only 2%.

**Sensitivity Analysis.** To determine the sensitivity of the optimal SMU size to user-selected parameters, the cutoff grade and reference polygon were varied and ore/waste tonnages recalculated for this example.

The cutoff grade was varied from a low of 3% to a high of 7%, and the results of SMU size vs. tonnage/grade were used to calculate an optimal size for each cutoff (Figure 8). The optimal SMU size based on grade had inconsistent results that were all very small compared to the SMU size based on tonnage. The optimal SMU size appears to reach a maximum around a 5% cutoff grade. The mean grade for all the ore is 5.211% and the median is 4.991%. It appears the optimal SMU size is largest when the cutoff is near these statistics; however, other cases would have to be examined to confirming this relation.
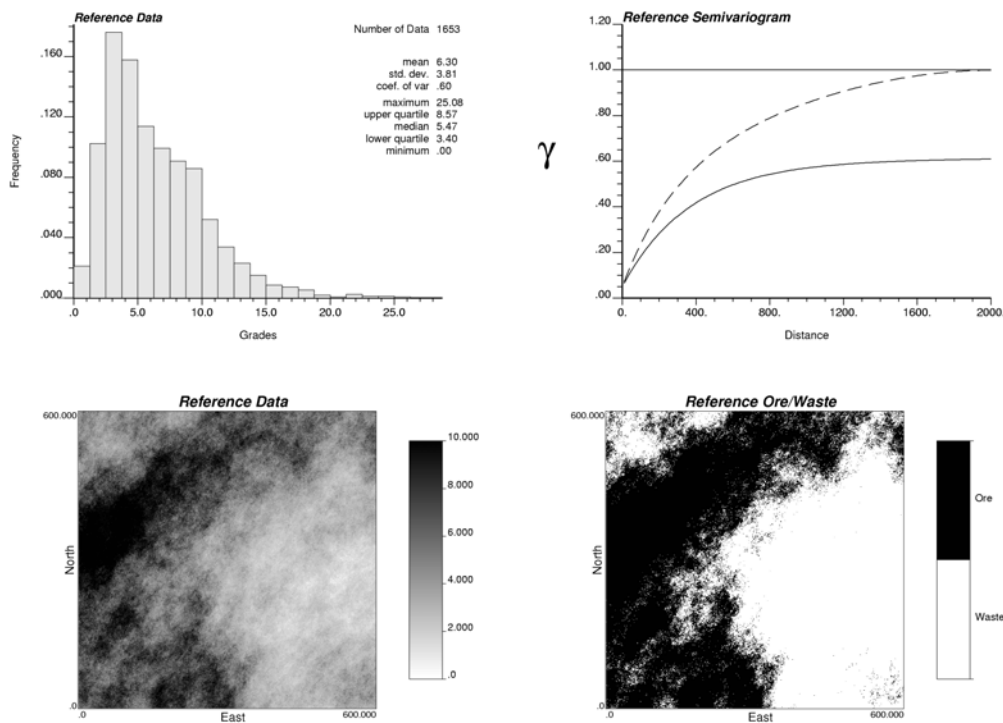


**Figure 2. Reference data: histogram (top left), variogram (top right), map of reference data (bottom left), and the map of the reference ore-waste classification (bottom right).**

To analyze sensitivity of the SMU selection to the polygon used for determining the reference values, initial polygons digitized by seven different people were used to obtain different reference values. An semi-automatic dig limit optimization program was also used to find a polygon (Norrena and Deutsch, 2002). Ore tonnages were used to determine the optimal SMU sizes because of the larger spread in the values. The different reference values are shown along with actual recovery at a cutoff grade of 5.0% in Figure 8. The resulting optimal SMU sizes range from below 5m to over 50m, suggesting that the results of this kind of study would be better used for comparison rather than obtaining absolute values. Also, the results using different cutoff grades must be done by the same person to give consistent results. Another alternative to obtain consistent results would be to use an optimization software such as the one used here.
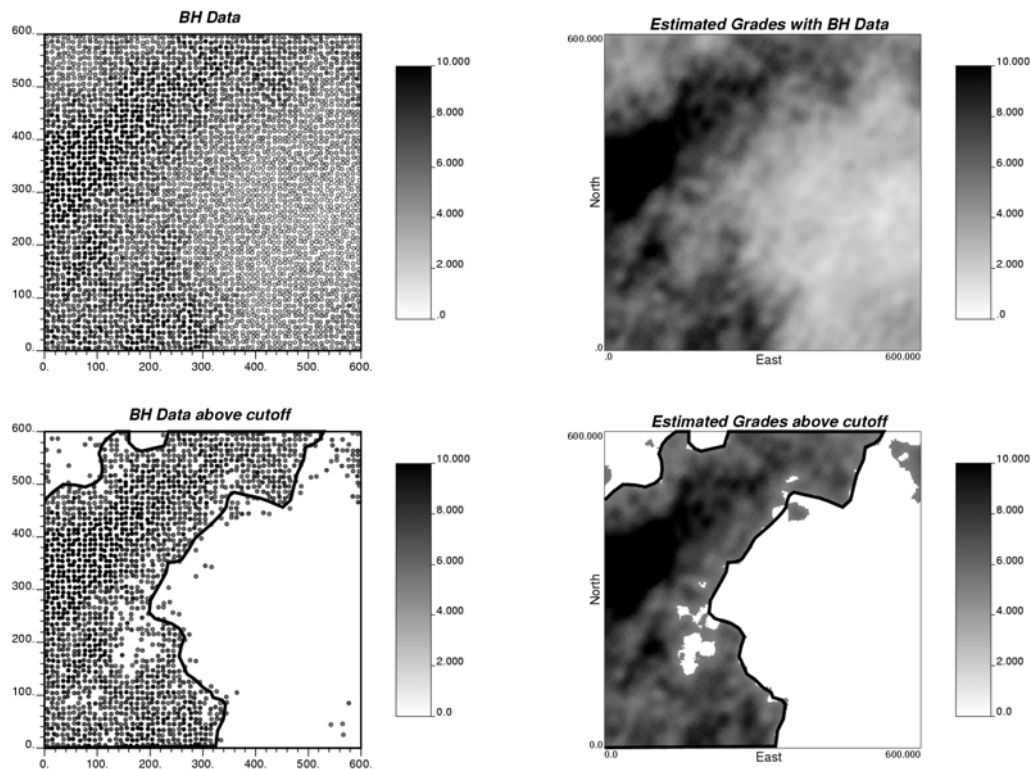


**Figure 3. Maps of: (1) blasthole (BH) samples at nominally 10m x 10m spacing (top left), (2) estimated grades using ordinary kriging all BH data (top right), (3) only those BH samples above the cutoff grade of 5% (bottom left), and (4) only those estimated grades above the cutoff grade (bottom right). Outline based on the estimated grades map is shown in (3) and (4).**
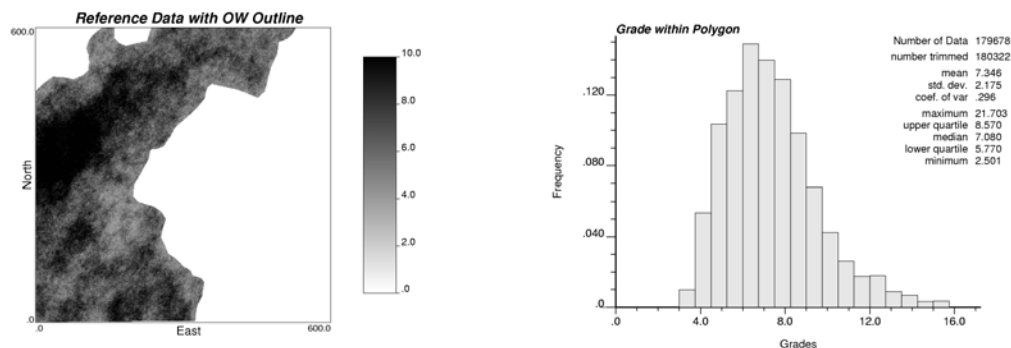


**Figure 4 Map of reference grade for ore (left) and histogram of grades within the ore polygon (right).**
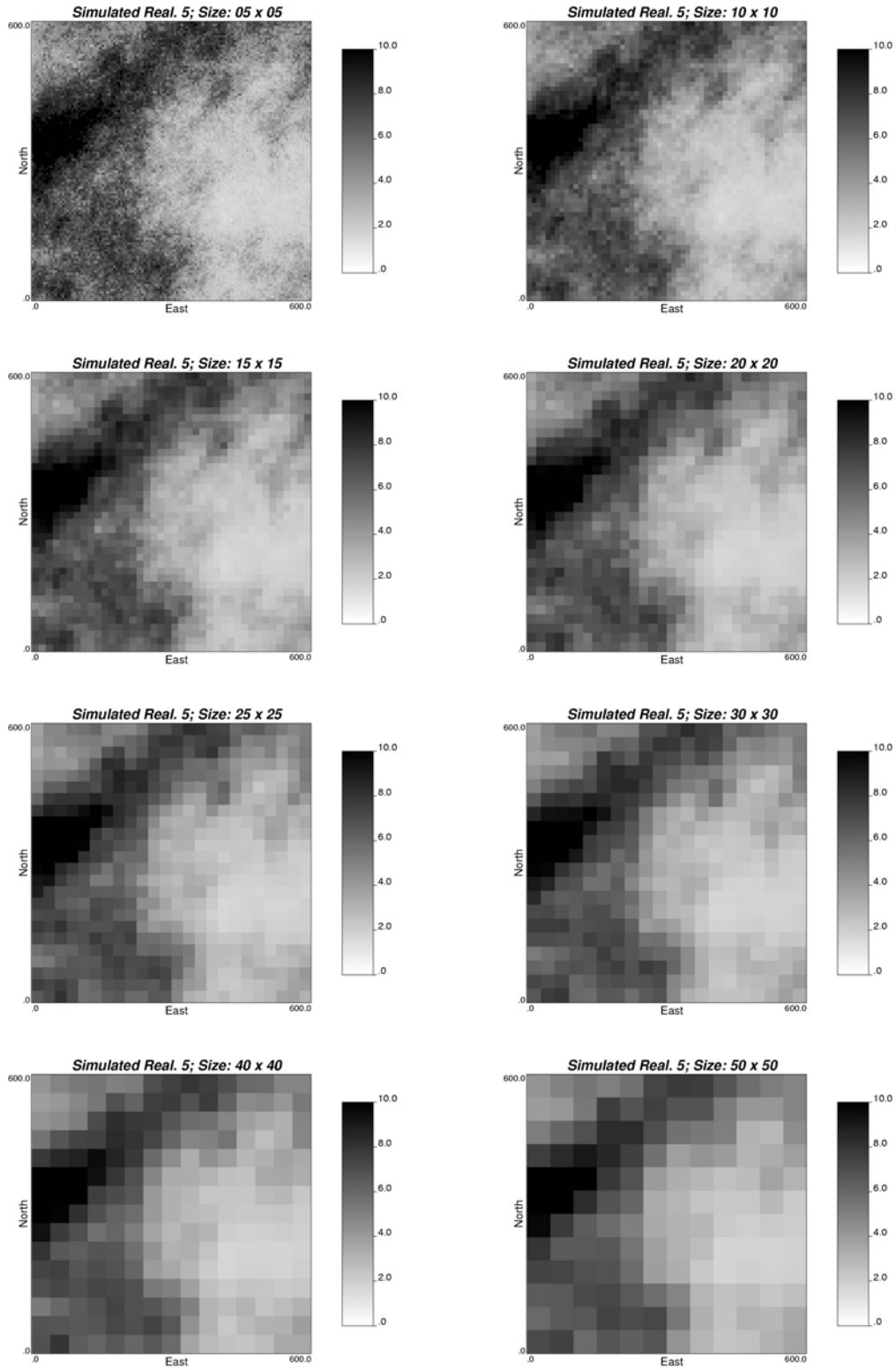
**Figure 5. One realization from conditional simulation at 5m x 5m blocks (top left). This realization is block averaged to seven different SMU sizes: 10x10, 15x15, 20x20, 25x25, 30x30, 40x40, and 50x50.**
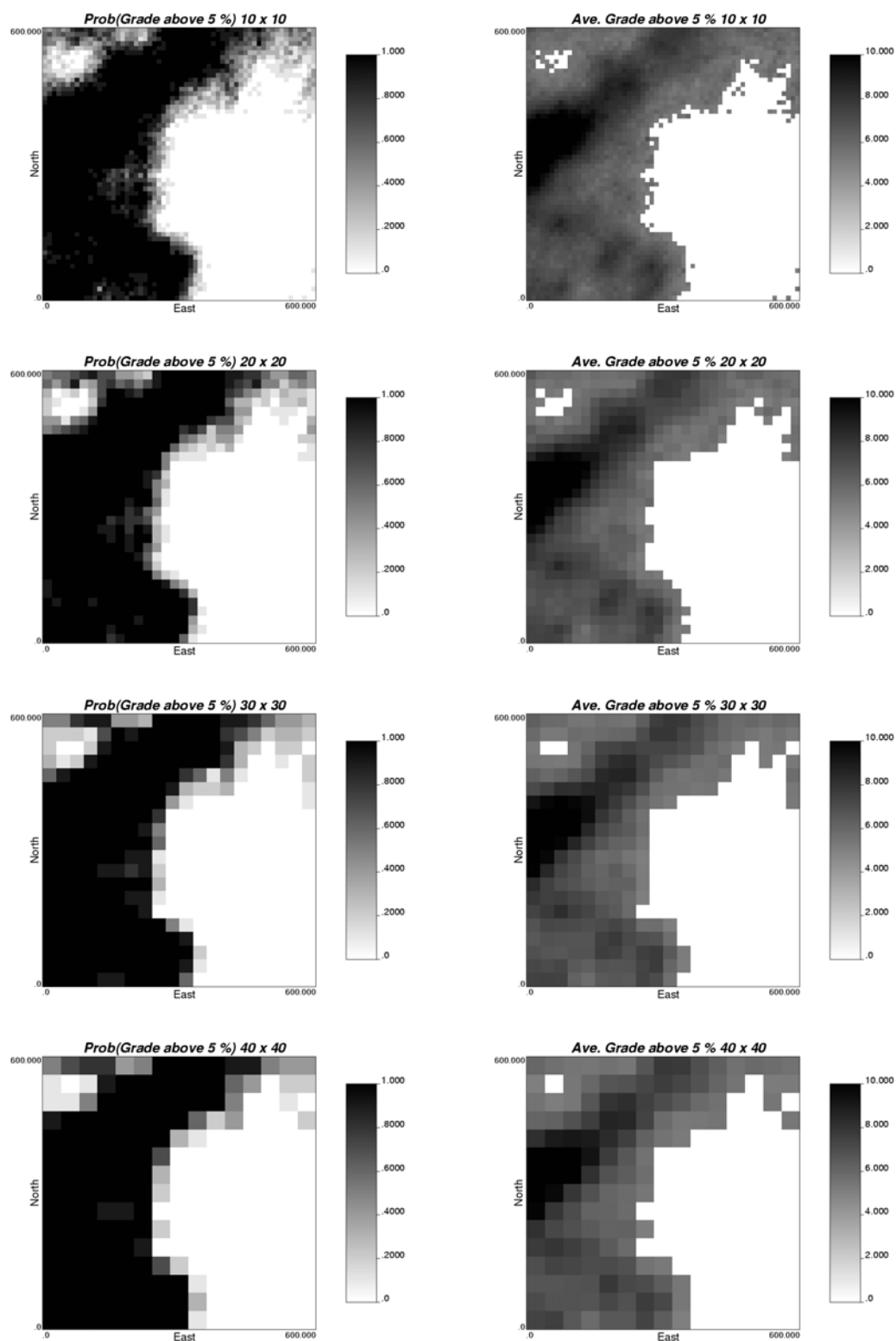
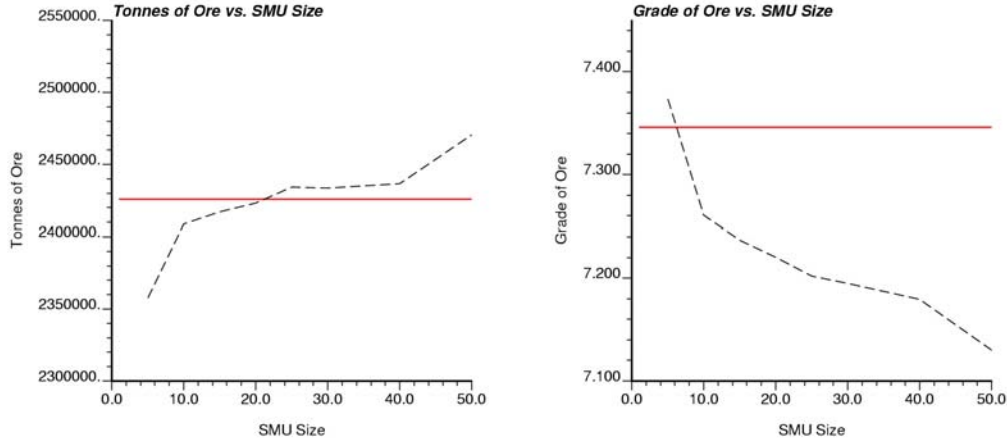**Figure 6 Probability (left) and grade (right) of ore maps for SMU sizes: 10x10, 20x20, 30x30, and 40x40.**

**Figure 7. Graph of tonnes of ore (left) and grade of ore (right) versus SMU size. The reference values are plotted as solid, horizontal lines.**
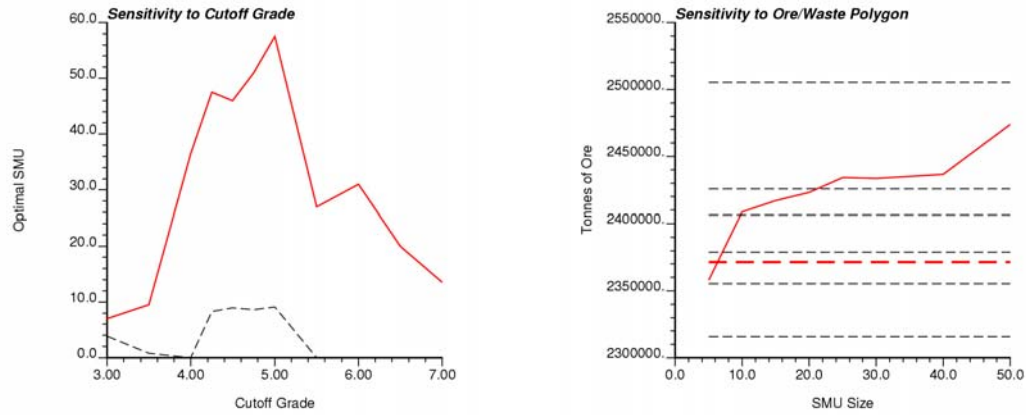


**Figure 8. Sensitivity of optimal SMU size to cutoff grade (left) and to ore waste polygon used to determine the reference tonnes of ore (right). For the sensitivity to cutoff grade, the solid line corresponds to the optimal SMU based on tonnes, while the dashed line corresponds to the optimal SMU based on grade. For the sensitivity to the polygon, the solid line shows the relation of ore tonnage to the SMU size, while the dashed lines represent the reference tonnes of ore as determined by different ore/waste limits. The thickest, longest dashed line corresponds to the polygon obtained from the semi-automatic dig limit optimization program.**

## Discussion

In certain instances, an analytical approach could also be taken to determine the appropriate SMU size. For example, if the distribution of tonnes is known to be normally distributed with mean $\mu$ and standard deviation $\sigma^2$, then the relation between the recovered tonnes given a cutoff grade can be calculated as (Journel and Huijbregts, p. 480) (a similar type of function is also available for a lognormal case):

$$T(z) = T_o\left[1 - G\left(\frac{z - \mu}{\sigma}\right)\right] \tag{1.1}$$

Historical production data provides input information regarding the cutoff grade, the recovered and total in situ tonnes, as well as the mean grade of the processed material. Rearranging Equation (1.1), the standard deviation of the ore tonnage is given by:

$$\sigma = \frac{z - \mu}{G^{-1}\left(1 - \frac{T(z)}{T_o}\right)} \tag{1.2}$$

Thus, in order to match tonnes of ore, the appropriate SMU size would have a dispersion variance equal to the square of Equation (1.2), that is $D^2(v, A) = \sigma^2$. We also know that

$$D^2(v, A) = \overline{\gamma(A, A)} - \overline{\gamma(v, v)} \tag{1.3}$$

where

$$\overline{\gamma(A, A)} = \frac{1}{A \cdot A} \int_{A(\mathbf{u})} \int_{A(\mathbf{u'})} \gamma(y - y') dy'$$

and

$$\overline{\gamma(v, v)} = \frac{1}{v \cdot v} \int_{v(\mathbf{u})} \int_{v(\mathbf{u'})} \gamma(y - y') dy'$$

Given the variogram model and the size and geometry of the domain of interest, the calculation of $\overline{\gamma(A, A)}$ can be obtained numerically. If the domain can be considered ergodic, then $\overline{\gamma(A, A)} = \sigma^2_{data}$. The value of $\overline{\gamma(v, v)}$ is given by rearranging Equation (1.3): $\overline{\gamma(v, v)} = \overline{\gamma(A, A)} - D^2(v, A)$.

In practice, $\overline{\gamma(v, v)}$ is calculated numerically be discretizing the volume into point locations and the average variogram is calculated over these discretized points. This type of calculation requires knowing the volume over which the variogram must be averaged, which is unknown. For a simple rectangular block with a square base and known height, the sides of the square base can be determined by using the auxiliary functions and charts provided by Journel and Huijbregts (p. 112, 129). Charts are given for the case of a simple, isotropic spherical and exponential variogram. For instance, given that the variogram is spherical with a range of 200m, the bench height is 25m, and $\overline{\gamma(v, v)}$ =0.3, then the ratio of a side to the range (*l/a*) is approximately 0.38 (Journel and Huijbregts, p. 129). The size of the SMU is calculated as 0.38*200 or 76m x 76m.

Although charts and auxiliary functions can be used, many simplifying assumptions about the distributional shape, block geometry and even the spatial continuity are required to determine the SMU size analytically. The proposed methodology is more flexible in that simplifying assumptions are not required, and it can be applied to any deposit.

The results of applying this methodology shows that the SMU size does not need to correspond directly with the smallest mineable volume chosen by the mining engineer. Different SMU block sizes account for the fact that dig limits are drawn for grade control so a portion of the field is not mined for ore. The assumption of perfect selection combined with dilution from larger blocks can yield results that match the short term mine practice of selecting only a portion of the field and using samples that are closer together.

The underlying simulation models are generated at a fine scale, consistent with the

support of our drillhole samples. These models are always averaged up to larger scales for the purposes of ore reserve estimation. A big advantage of geostatistical simulation is that we can consider multiple SMU sizes (based on this exercise) to match the mill's production.

It will not always be possible to match both the tonnes of ore and the grade of ore that would be produced by the mill. The graphs obtained from the example shows that the optimal SMU size depends on whether we are matching tonnes or grade. The tonnes of waste is also an issue, but this is inversely related to the tonnes of ore so consideration of tonnes of ore automatically accounts for the tonnes of waste.

Selection of the appropriate SMU is a compromise between getting the right estimates for tonnage of ore and getting the right grade for this material. This compromise depends on the magnitude of difference associated with selecting one SMU over the other, which amounts to determining (1) whether the discrepancy between the actual and estimated grade is too significant to accept even though the tonnage is correct, or (2) whether the discrepancy between the actual and estimated tonnage is too large to accept even though the grade of ore is correct.

It is possible to obtain a graph where the simulation results at different SMU sizes intersects the reference value at multiple sizes. Figure 9 shows an example of one such case. More studies are required to sort all of these details out. Our goal could be to match metal content.  Another concern is that the simulation results may never intersect the reference value (Figure 10). This is related to the available data - there may be too few samples to obtain "good" simulations. The distribution of sampled drillhole data may be inconsistent with the distribution of more closely spaced blasthole grades. Considering multiple realizations and/or discarding realizations that are too inconsistent would make the results more stable.

Although ore/waste limits may be drawn by the mine geologist, some dilution of the delineated zone is expected due to site operations such as errors in survey staking, and sloughing of blasted material. These result in dilution due to mine operations. There are a number of ways that we could account for this type of dilution. The initial ore/waste limits identified in conventional grade control practice can be dilated and eroded to account for these forms of operational dilution. The tonnes and grade of ore can then be calculated using these modified ore/waste outlines. Comparison with the simulation results would then be checked using these revised estimates.  The selection may also be improved by on-site visual refinements by the geologist or equipment operator. This is more difficult to account for in a simulation context.

External dilution due to sharp geological boundaries is another important consideration. These contacts may distinguish between the mineralized host rock and barren rock; thus, poorly delineated ore/waste contacts can result in a significant amount of dilution. In these cases, more exploration drilling may be required to identify these regions. Simulation of two different populations is recommended to obtain more reliable reserve estimates and, consequently, better SMU size selection.

**Final Comments**
This paper gives some procedures that are useful for SMU block size selection. There remains a great deal of subjectivity and site-specific considerations. It would be very useful to calibrate the results to actual production in an operating mine. Ore reserve estimation over production periods can be refined by performing this type of exercise over already mined areas. This would involve determining the appropriate SMU size from previous production period(s) and using this SMU to forecast the next production period.
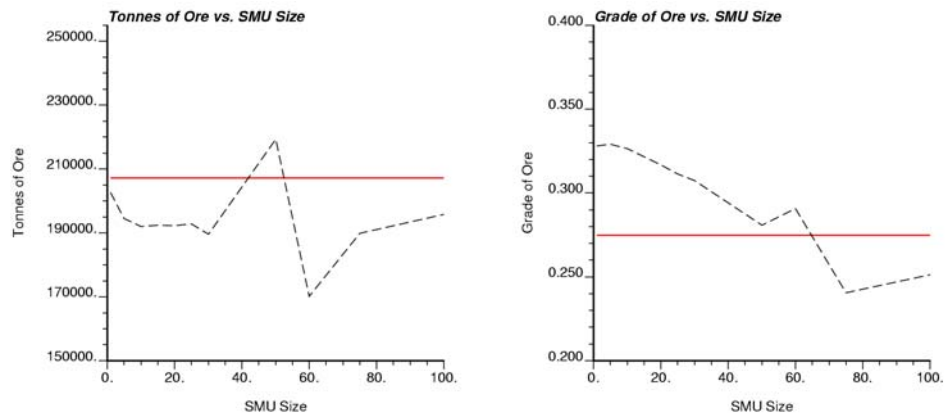
**Figure 9. Example of multiple optimal SMU sizes from graphs of tonnes of ore (left) and grade of ore (right) versus SMU size. The reference values are plotted as solid, horizontal lines.**
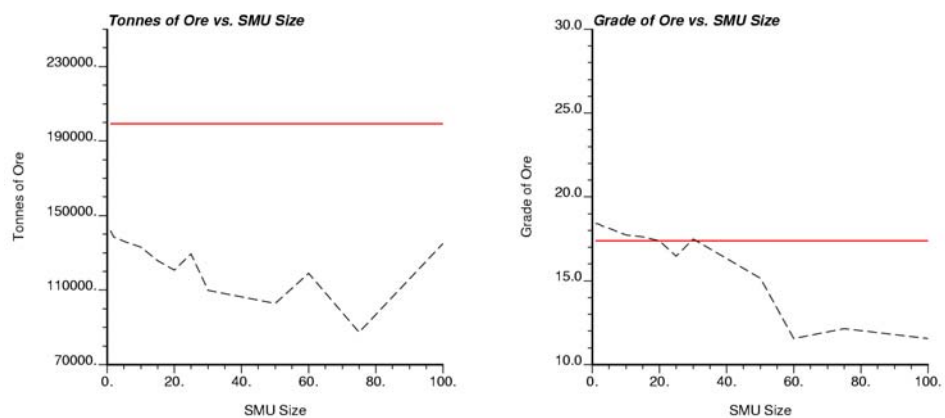


**Figure 10. Example of non-intersection of graph of tonnes of ore (left), and multiple intersection of grade of ore (right) versus SMU size. The reference values are plotted as solid, horizontal lines.**

There are many details related to how the blastholes are sampled, how geological boundaries are handled, how the different types of dilution intervene in the resource calculations, how multiple metals and contaminants are handled and so on. Additional work is warranted to study the sensitivity to these considerations.

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